New methods for investigating mode coupling in multimode fiber: Impact on high-speed links and channel equalization

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Introduction

Multimode fiber has reemerged as a potentially high performance optical transport media with under exploited capabilities. By examination of the impulse response of MMF with large temporal resolution and large dynamic range, enabled by measurements with sub-picosecond jitter, we quantify the impact on the impulse response and determine the mode coupling. These results together with numerical models enable us to accurately depict the transmission of ultrafast data streams, and thereby quantify system performance increases in the context of adding optical and electronic signal processing to legacy and modern multimode fiber.

We first provide an overview of the mode coupling theoretical results, which lead to the fiber impulse response for various levels of coupling strength. This provides a motivation for the new experimental methods which are subsequently discussed. The experimental results are then fit to the mode coupling theory, providing a measure of mode coupling strength in modern multimode fiber. We then discuss the impact of mode coupling on the performance of electronic equalization methods. Lastly we outline our new results which exploit Raman gain to selectively amplify only one mode group and thereby improve the fiber impulse response.

Mode coupling theory

A set of coupled equations describing the power flow in a waveguide with N modes can be derived via a perturbation approach [1].

$$\frac{\partial P_n}{\partial z} + \tau_n \frac{\partial P_n}{\partial t} = -\gamma_n P_n + \sum_k d_{n,k}\left(P_k - P_n\right) \tag{1}$$

where, P_n is the power in the n^{th} mode at time t at a distance z. γ_n is the attenuation coefficient for nth mode and $d_{n,k}$ is the coupling coefficient between two modes n and k. In deriving (1) a number of assumptions are made: a) the imperfections in the waveguide such as changes in diameter, elliptical core deformations, or random bends of the axis lead to coupling of modes; b) these imperfections appear in a random fashion along the length of the waveguide and a statistical ensemble average can be taken; c) the waveguide perturbation is uncorrelated to the field amplitude beyond a certain correlation length; d) the distance required to create any change in field amplitude is large compared to correlation length. The assumptions c) and d) essentially make the waveguide perturbation function and field amplitudes uncorrelated over the entire waveguide. It should be noted that the perturbation interacts strongly with the phases of the field and the effect is integrated over the length of the waveguide. The spatial frequencies of the perturbation are determined via a Fourier analysis and are used to quantify the interaction with the propagation phase difference of the modes. The contribution is contained in the mode coupling coefficient $d_{n,k}$ given by

$$d_{n,k} = A \cdot \left[\frac{\left(\pi k_0 c \varepsilon_0\right)^2}{8} \cdot \rho_{n,k}^2 \cdot \frac{1}{\left(\Delta\beta_{n,k}\right)^8} \right] \tag{2}$$

$$\rho_{n,k} = \int_0^\infty r E_n(r) E_k(r) \frac{\partial \tilde{n}^2}{\partial r} dr \tag{3}$$

where, E_n is the radial electric field profile of the n^{th} mode, n(r) is the refractive index profile, $\Delta\beta_{n,k}$ is the difference in propagation constant of mode n and k and A is the mode coupling strength. The electric field profiles and the propagation constants as well as the group delays used later are all numerically determined using our mode solver which allows arbitrary core index profiles. The electric field profiles in (3) are normalized such that:

$$\int_0^{\infty} rE_n^2(r)dr = \frac{k_0}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{1}{\beta_n} \tag{4}$$

There are wave vector degeneracies among the modes and complete mode coupling is assumed between the these modes which form a mode group as suggested by (2). Furthermore, since the adjacent mode groups have the smallest $\Delta\beta_{n,k}$, coupling is dominated by them. Considering this the average power P_g of the mode group g is found by solving the equations:

$$\frac{\partial P_g}{\partial z} + \tau_g \frac{\partial P_g}{\partial t} = \begin{cases} -\gamma_g P_g + d_g\left(P_{g+1} - P_g\right) \qquad + \left(\frac{g-1}{g}\right) d_{g-1}\left(P_{g-1} - P_g\right) & \text{for } g \text{ even} \\ -\gamma_g P_g + \left(\frac{g}{g+1}\right) d_g\left(P_{g+1} - P_g\right) + \left(\frac{g-1}{g+1}\right) d_{g-1}\left(P_{g-1} - P_g\right) & \text{for } g \text{ odd} \end{cases} \tag{5}$$

The coupling coefficient d_g between two adjacent mode groups g and g+1 can also be found using an approximate analytical solution [2] and we find that it matches well with the exact value obtained using (2), (3) and (4) directly

$$d_g = \frac{1}{2}\left(\frac{nk_0}{a}\right)^2 \Delta^2 \left(\frac{g}{g_{TOT}}\right) \frac{A}{\left(\Delta\beta_{g,g+1}\right)^8} \tag{6}$$

where, g_{TOT} is the total number of mode groups present and a is the radius of the fiber. A split step method [3] is used to evaluate (5) for a range of mode coupling strengths A.

Figure 1 depicts the impulse response for a 62.5μm multimode fiber (MMF) subject to a center launch Gaussian spot excitation consistent with the mode size of single mode fiber. The intrinsic bandwidth is that of a 10Gbps channel. The nature of the channel response is strongly dependent on mode coupling. For $A < 10^4$ the coupling has negligible effect and we observe the arrival of the distinct mode groups. On the other hand, for $A \geq 10^{12}$ the response becomes Gaussian due to strong mode coupling. For intermediate coupling, energy is seen to "fill in" the valleys between the primary mode groups. Essentially, some energy propagates with a group velocity correspond to some average of other distinct groups.

This observation suggests the new experimental method; observe the impulse response with sufficient temporal resolution and dynamic range to observe this fill-in effect. Then the coupling coefficient can be estimated by fitting the results to the numerical model.

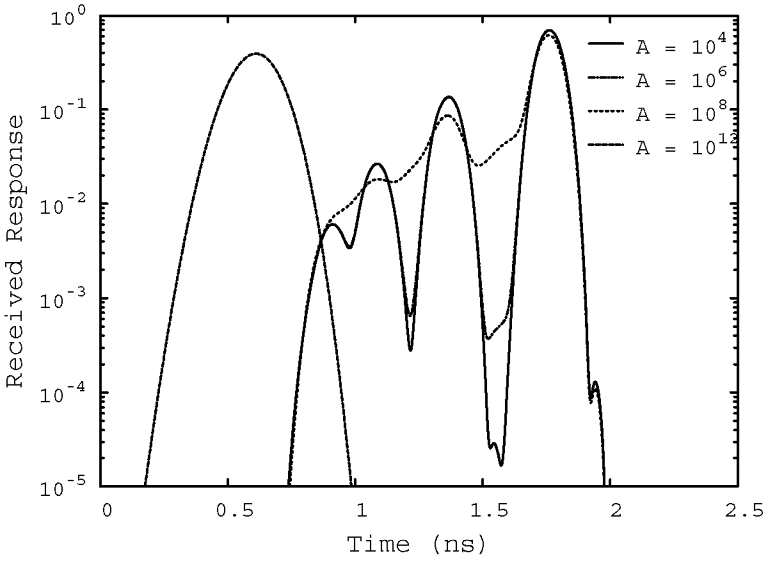

Fig. 1. Numerical estimate of the effects of mode coupling on received pulse shape for an 8km MMF. For weak coupling ($A \leq 10^4$), the arrival of distinct mode groups is observed. For strong coupling ($A \geq 10^{12}$), the received pulse is nearly Gaussian. For intermediate coupling, energy exchange between the modes results in energy arrival at times other than that of the distinct mode groups and the "valleys" are gradually filled in. [Ref. 3]

Measurement methods

Previous methods to determine the mode coupling strength in (MMF) are essentially based on comparing the evolution of mode power distribution (MPD) observed experimentally with that predicted by the mode coupling theory [1]. The far field analysis method is used to determine the mode power distribution at the end of the fiber. Our method relies more on comparing the evolution of temporal impulse response of the fiber with that predicted by a MMF model including the mode coupling. Apart from the change in MPD, the other obvious effect of mode coupling is the energy filling between the distinct mode groups and the corresponding change in the temporal response. The MPD at launch is dependent on the excitation and the specific fiber and is not directly measurable. Therefore, instead of comparing the change in MPD, we focus on the impulse response, which is sensitive to the mode coupling coefficient and, more importantly, is directly related to the channel performance.

Impulse response measurements of the MMF are done with sufficient temporal resolution and dynamic range to separate the mode groups distinctly and quantify the energy arriving between them. To minimize the chromatic dispersion, 16 ps FWHM pulses at 1550 nm are launched. A mode locked laser followed by bandpass filters is used to generate the nearly transform-limited pulses. A detector-sampling module with net bandwidth of 20 GHz is used as the receiver. A GRIN lens couples all modes of 50 μm MMF in the detector with nearly 100% efficiency. The signal is launched into the MMF core with a single mode fiber (SMF) at different offsets which allows control of the power distribution in the excited modes. To reduce the noise, the response is effectively averaged for 32 minutes and 3 orders dynamic range is obtained. Care is taken to minimize the jitter and drift in the acquired waveforms.

The use of 1550 nm helps to insure that the primary modes are temporally separated due to large differential modal delay (DMD) at 1550 nm for fibers optimized at 1310 nm. Coupling strength A, which is a function of micro-bending perturbation and mechanical properties of fiber, is expected to be only slightly dependent on the wavelength of operation. By comparing the measured impulse responses of 4.4 km 50 μm MMF with numerically generated impulse response we estimate a coupling strength A of $1\text{x}10^8$. The observed uniformity of arrival of the mode groups suggest that the fibers examined can be described by an alpha profile [4] without any major index profile defects. Hence, in the numerical model, we assume the $\Delta\beta$'s to be that of a fiber with a pure alpha index profile. The modal delays and the mode power distribution at the launch are adjusted for different correlation length to match the received waveform. We also estimate the coupling strength A of typical 62.5 μm FDDI grade fiber to be $5\text{x}10^7$.

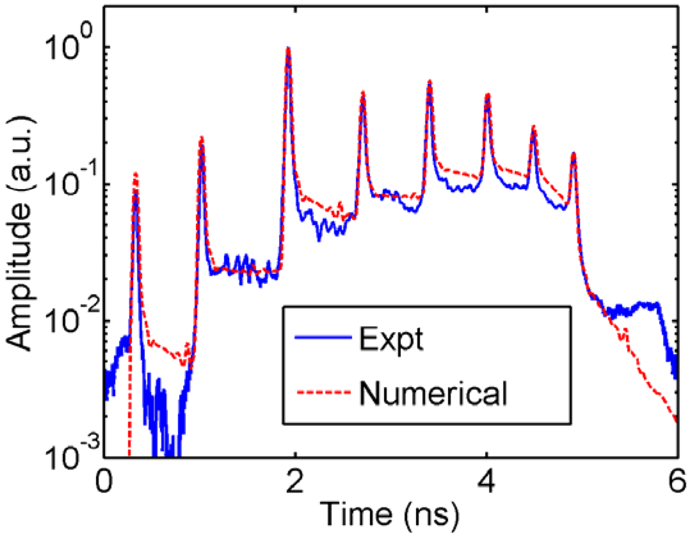

Fig. 2. Comparison of measured MMF response and the numerical estimate using a mode coupling strength of $A=1\text{x}10^8$. Energy arriving between the primary mode groups is due to mode coupling. For this fiber, at 1550nm, the lower order modes arrive later. The lower order modes are seen to exhibit lower coupling compared to the higher order mode groups as predicted by the theory.

The method described above is specifically suitable for MMFs with relatively low mode coupling strength and therefore, requires relatively long fiber lengths in order to observe the effects of coupling. In that regime, modes can be resolved temporally and the estimation of mode coupling strength is accurate and the impulse response is useful in estimated the efficacy of mitigation methods. We note that for smaller lengths of fiber, the temporal resolution may be made sufficient by using shorter launch pulses and a receiver with larger bandwidth. However, the dynamic range of the measurement is often limited by the tail of the detector response and an appropriate length of fiber should be chosen to clearly observe the mode coupling effects. Nonlinear methods of measuring optical pulses such as autocorrelation and cross-correlation may also be used. However, the nonlinear interaction between different modes needs to be quantified and hence extracting the information quantitatively from the observed response is challenging.

In plastic optical fibers (POFs) strong mode coupling is observed and complete mode coupling is observed within 10-30 m of fiber length [6]. A method suitable in this regime illuminates the fiber end at an angle to preferentially excite higher order modes (HOMs) [7,8], mode coupling changes the MPD to couple power into the lower order modes (LOMs). The far field patterns of LOM and HOM are disk-like and annular-shaped respectively. For an appropriate length of fiber, where mode coupling effect is intermediate, transition of far field image from disk to annular shape occur at a particular angle. From the measured angle and fiber length, the mode coupling strength can be found to a first order approximation. The extracted mode coupling strength can be used in the numerical model for verification and prediction. Figure 3 illustrates the impulse response of POF over 200m, no structure is observed, indicative of the strong coupling regime. However, the response is suitable for >10Gbps links.

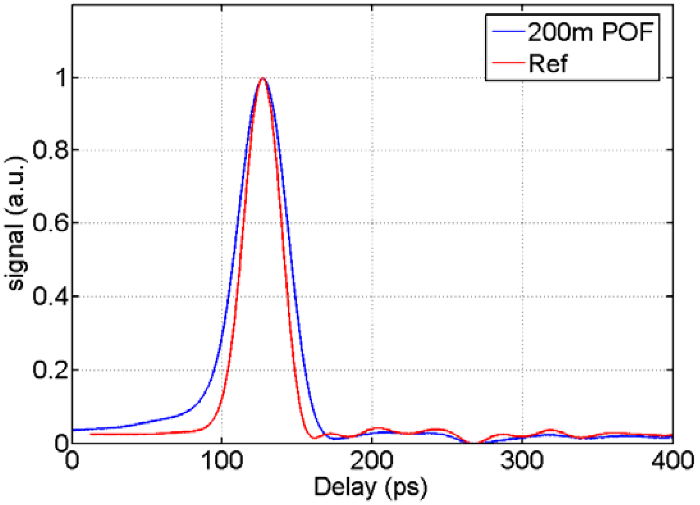

Fig. 3. Response of 200 m of 50 μm core perfluorinated polymer based POF (GigaPOF-50SR from Chromis Fiberoptics). Absence of any modal structure and nearly Gaussian response indicate a strong mode coupling regime.

Electronic Equalization Analysis

Different electronic equalization methods to mitigate channel impairments are extensively used for copper and wireless links. For 10 Gbps data transmission via optical channels less complex structures such as feedforward equalizer (FFE) and decision feedback equalizer (DFE) architectures are implemented. FFE, which is essentially a linear filter, attempts to invert the channel and in the process necessarily increases the noise-to-signal ratio of channel. DFE partially avoids the problem by removing the post-curser inter symbol interference (ISI) estimated from the previously detected bits, using a feedback filter. However, for both cases the equalization process incurs a signal-to-noise (SNR) penalty which can be translated to an optical power penalty. Though the equalization penalty depends on the specific filter architecture, number of filter taps etc., the ideal equalization penalty for infinite length DFE (PIE-D) can be computed which provides a lower limit for the penalty of specific implementations [5].

We compute the ideal equalization penalty for the 10 Gbps, 62.5μm MMF links with and without mode coupling for different fiber lengths and two launch conditions: center launch (0-3 μm offset) and offset launch (17-23 μm offset). The range of each launch condition accounts for the fiber coupler misalignment tolerance and the offset launch is under consideration as a part of an IEEE standard [9]. Based upon the measured results a coupling strength $A = 5\text{x}10^7$ is assumed. We examine two fiber index profiles, a pure alpha profile and a profile characterized by a dip in the index on center. This irregularity is common in some legacy MMF.

Figure 4 depicts the equalizer penalty for these cases and shows that coupling systematically increases the residual equalizer penalty. Additionally, the increase depends strongly on fiber index profile. This can be understood by first considering the equalizer penalty in the absence of coupling. Initially, the penalty linearly increases with fiber length. This results from the temporal spreading of the mode groups. As the energy slowly spreads outside of a single bit slot the penalty increases. After some fiber length only the dominant mode group is left in one bit slot and all others mode groups contribute to ISI. In this regime the infinite equalizer penalty no longer increases. The transition point depends on the relative DMD of the fiber and since the center dip has larger DMD it shows that it has already reached this saturated penalty regime by 300m for center launch, Fig. 4a.

Now consider the action of mode coupling. First we note that the DMD dominates the equalizer penalty. However, once the saturated penalty regime is reached coupling increases the penalty, in some cases by substantial amounts. This results from the slow coupling of energy out of the dominant mode group into the other groups which contribute to ISI. The center dip profile is more susceptible to this due to the larger mode coupling found with center dip profiles. Similarly, the effect of mode coupling is larger for offset launch, Fig. 4 b since the higher order modes, which are preferentially exited with offset launch, exhibit larger coupling than the lower order mode groups.

We note that though the general observations remain unchanged, larger coupling strength as measured for 50 μm MMF will have larger impact on the equalization penalty.

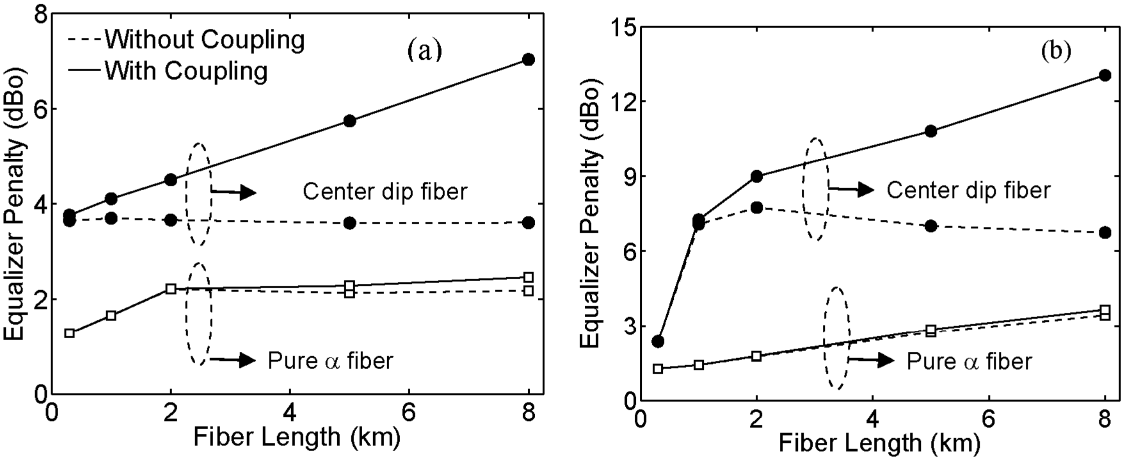

Fig. 4. Impact of mode coupling on equalizer penalty as the fiber length is increased: (a) Center launch (b) Offset launch. For each launch condition, both the pure alpha and center dip fibers are shown [4].

Raman amplification in MMF

We use Raman amplification in MMF as an optical signal processing technique to boost the signal strength as well as improve upon the signal integrity. Numerical calculations show that Raman gain of the lowest order mode LP_{01} in 62.5 μm MMF is comparable to that of SMF. This results from the larger GeO2 concentration in the core and hence the larger Raman gain coefficient which offsets the somewhat larger effective area of the lowest mode group compared to SMF [10]. Furthermore, the larger effective area of the higher order modes reduces the Raman gain of the higher order modes compared to that of LP_{01}. This enables Raman gain to selectively amplify the LP_{01} mode and thereby reduce the inter symbol interference from higher order modes. We experimentally show for co-propagating pump set up and for near-center launch with a SMF, the response of MMF is improved via mode selective gain.

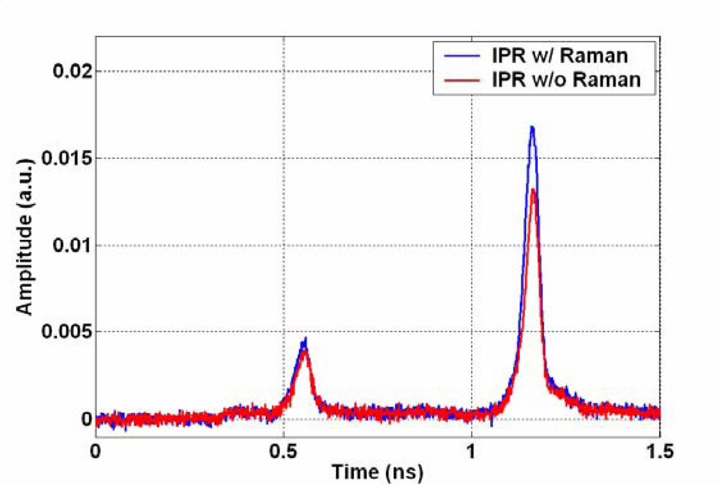

Fig. 5. Improvement in the impulse response via mode selective Raman amplification. 200 mW of 1450 nm pump and 16 ps pulses at 1500 nm are launched in 9 km of 62.5 μm MMF via SMF at 3 μm offset from the center of the core.

Conclusions

We have shown that mode coupling effects in multimode fiber can be readily observed and quantified using a new high temporal resolution, high dynamic range impulse response characterization method. The effects of coupling are shown to increase the power penalties for subsequent electronic filtering. We also demonstrate an optical signal processing method using mode selective Raman gain that effectively reduces intersymbol interference and enables high speed long reach MMF links.

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