Ollie Rabbit in the Kingdom of Arithmetic Coding

Once upon a time in the kingdom of Arithmetic Coding there lived a little rabbit named Ollie who lived with his family in a small village. Their small village would always eat well as the villagers knew where the best berries and nuts were to be found in the nearby forest The villagers also stored their food alongside certain paths in the forest.

But like all fairy tales, there was a mean and evil person trying to trick them. In this story, there was an evil royal taxman that wanted to find out where the villagers collected and stored their food. Unfortunately for him, there was a good but simple king who would not allow him to hurt the villagers and tell him where their food was stored. So the evil taxman had to try and figure out a sneaky way to discover where they were going in the forest.

Fortunately, for the taxman, the paths that lead away from the village into the woods were unique in that as you went down any path it would always split into two paths.

The Forest Map
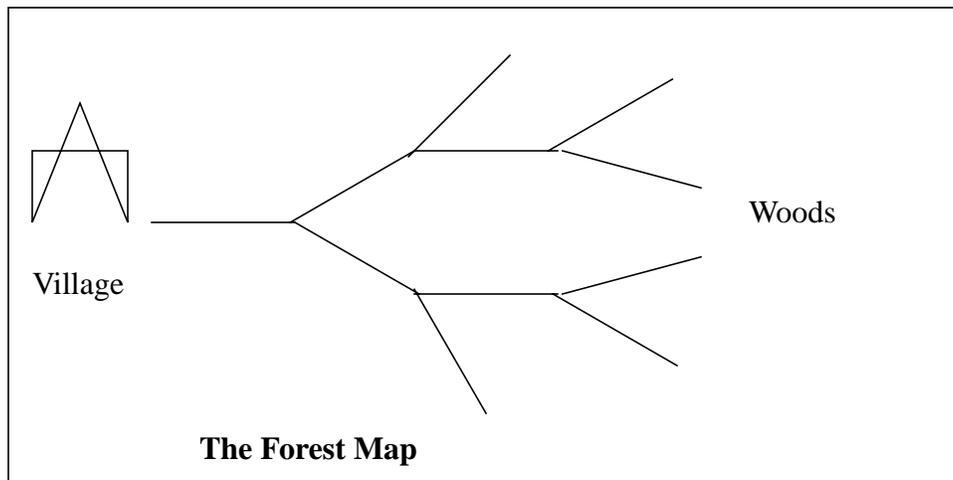
The evil taxman decided that to find out where the villagers went he would station a guard at every split in the path. That way he would find out where they went by simply asking the guards. But the nice king seeing what the taxman had planned said, "This idea is fine as long as you use the villagers as the guards at the splits." The evil taxman knew that he was foiled again. The villagers would never tell him where they were going!

So he tried to think of another way to find out where they were going. He decided to charge the villagers to enter the forest. That way he could figure out how far they went by how much it cost them to get where they were going. And he could still use the villagers as toll collectors! "This will get past the king", he thought.

So now he had to decide what the fees to charge. At first he thought, "I could start by charging them $50 dollars no matter which path they went down at the first split. Then charge them $25 dollars at the next split in the path, then charge them $12.50 at the next split and so on". This would be an easy formula. At each split in the path he would charge them half the value of the previous split.

But then he realized that he would never figure out where they went because each path in the forest would cost the same and he would not know which path they took at each split. So he decided to charge them according to how many people had taken the path.

He called the path that the most people went down the most probable path(MPP) and the other path, the least probable path(LPP). He knew that at each split in the path that a 3/4 of the villagers went down the most probable path and a 1/4 went down the least probable path at every split in the path. Even more fascinating was the fact that the right path was always the most probable path at each split in the paths (what a phenomenon!). "This was ideal for him", he thought. Now I can just split the fees 3/4 and 1/4 and I will be able to tell where they went by how much it cost them to go into the forest.

Map of Probabilities

Map of tolls
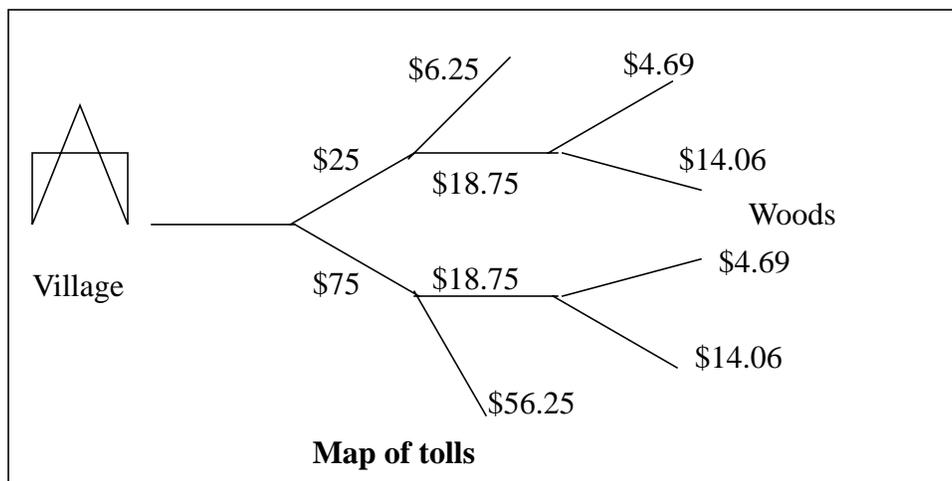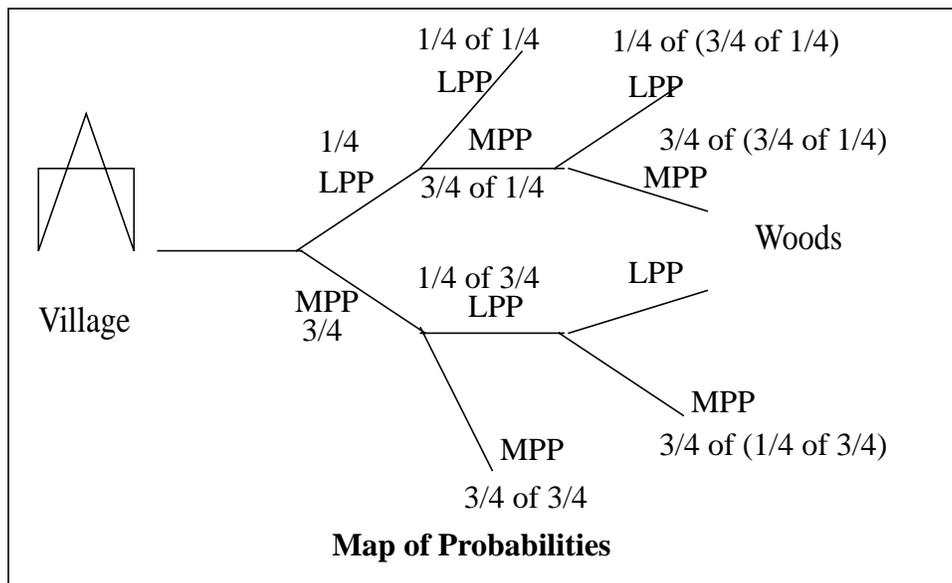
He told the king of his idea. The king thought a while and seeing that the taxman would find out where the villagers went said, "I don't like it. You are charging them too much money. No one should be charged more than $100 to go into the forest, no matter how far they go." Looking

at the map of tolls, the evil taxman knew that if a villager went down the most probable path twice in a row it would cost him $75 + $56.25 = $131.25, over the limit of $100.

The evil taxman scratched his head and the king went on, "I know what we will do. We will always make the less probable path free and charge the villagers going down the most probable path what it would have cost them if they had gone down the less probable path on your map of tolls. Go back to your quarters and draw me a new Map of Tolls." The evil taxman stomped back to his quarters thinking, "How am I ever going to find out where the villagers are going?"

The evil taxman decided not to go through with posting guards to collect the tolls as he would have to supervise the villagers and this would not get him anywhere. Instead the king had suggested that he go with the honesty system where the villagers would report how much it cost them to go in the forest. The evil taxman could not think of a way around this situation and he gave up trying to use the tolls as a way to tell where the villagers were going.

New Map of tolls
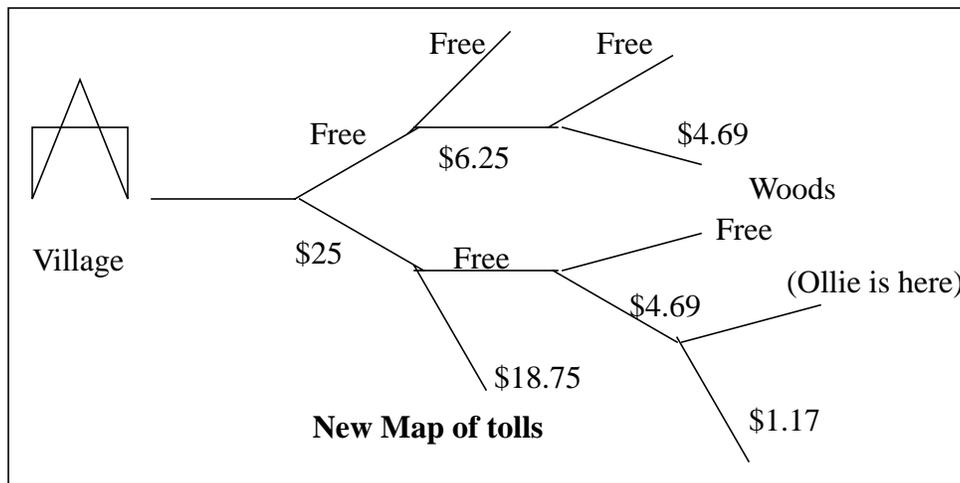
Now back to our friend Ollie. One day while he was going through the forest with his friend Stan, he wondered off the path and fell into a hidden pit. Now Stan was not too bright and Ollie knew that Stan would not remember all of the splits in the path that they had transversed once he got back to the village to get his lazy brother Renny, who was sleeping in the village, to come and help him get out. So he had to think of a way that Stan would remember to find him again when he came back with help.

Thinking for a while, he figured out how he could use the toll system and the number of splits in the path to get Stan right back to the correct path. So far it had cost him exactly $29.69 and he had gone through 4 splits in the road. But he thought that if Renny had to carry all the currency that made up $29.69 he would get tired and not make it. He thought again. Well, at the last split in the road if he went the other way it would have cost him $29.69 + $1.17 = $30.78. So he knew that he could use any value between $29.69 and $30.78. He decided on $30 and no change.

He knew Stan would have to remember two other pieces of information: how many splits in the path they had gone through (4), and to tell his brother Renny that he should always take the expensive (MPP) path if he can afford it.

Ollie knew that the number of splits that he had gone through was important because if his brother came down the path and did not know to look around on this particular path he could continue down the free paths forever.

Eventually, Stan made it back to the village and told Renny what his brother Ollie had said. So Renny and Stan got $30 and started off towards the forest. At the first split they came to they could pay $25 or take the free (LPP) path. Renny, remembering that his brother said to take the path if the could afford it, took the $25 (MPP) path. Now they only had $5 left. As they came to the next split in the path it would cost them $18.75 to take the expensive path but they did not have enough money so they took the free path. At the next split in the path, it was $4.69 for the expensive path and Renny took the expensive path. He now had 31 cents left over. When he came to the next split in the path, the expensive path was $1.17 and he knew that he could not afford it so he took the free path. He knew that since he had gone through 4 splits that he would find his brother somewhere along this path. Eventually, they found Ollie and went home.

Notes From the Author

As you can see the Map of Probabilities looks amazingly like Figure 1 in *Compression of Black-White Images with Arithmetic Coding*. Although this does not discuss encoding and the issues of Carry-over and keeping the A register normalized. It does discuss the issue of Decoding given only the number of symbols. It also gives some hint of encoding.

One can picture the model where the most probable path and less probable path is determined by decisions made in the past. So that if you had always taken the LPP it would eventually turn into the MPP at some split in the future. His brother would still follow the same path if they had done it this way since he would reproduce the most probable path and the least probable path as he repeated his brother's trail.

Another story could be told about how Ollie mapped all of the storage sites in the forest with the least amount of cash and change and the number of splits in the trail.