Turbulent Thermal Convection

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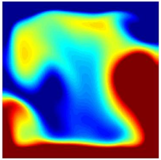

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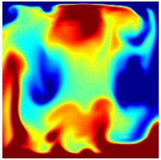
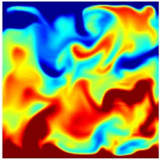
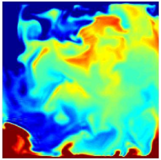
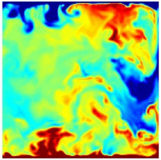


May 21, 2009

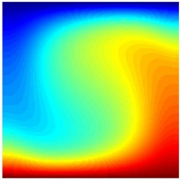




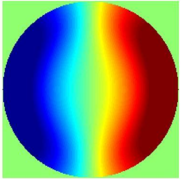


Rayleigh-Bénard Convection ($R \propto \Delta T$)

Steady-State $R = 2 \times 10^4$:

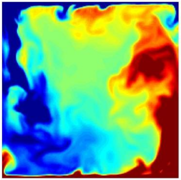


X-const

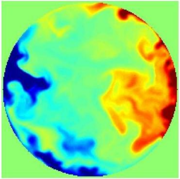


Z-const

Turbulence $R = 1 \times 10^8$:



X-const



Z-const

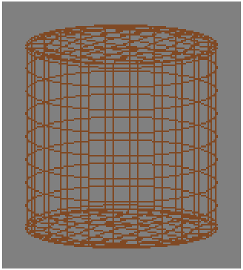
Dimensionless Boussinesq equations

$$
\begin{aligned}
\sigma^{-1}\left(\partial_t + \vec{u}\cdot\vec{\nabla}\right)\vec{u} &= -\vec{\nabla}P + \nabla^2\vec{u} + \theta\hat{z}, \\
\left(\partial_t + \vec{u}\cdot\vec{\nabla}\right)\theta &= \nabla^2\theta + Rw, \\
\vec{\nabla}\cdot\vec{u} &= 0,
\end{aligned}
$$

- $\vec{u}$ = (u, v, w) = velocity field, θ = temperature deviation field, P = pressure field
- ν= kinematic viscosity, κ= therm diffusivity, α = therm exp coeff
- R=Rayleigh number=$\alpha g \Delta T d^3/\kappa\nu$, σ=Prandtl number=ν/κ
- Γ = diameter/depth for cylinders

Numerical Simulations—Nekton

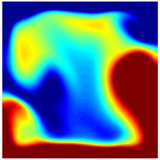
Numerically solves the Boussinesq equations



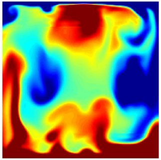
- P.F. Fischer—J. Comp. Phys **133**, 84 (1997)
  - Parallel, spectral element method
  - Gauss-Lobatto-Legendre grid
  - Semi-implicit: Treat off-diagonal terms explicitly:
    - Convective terms, Buoyancy term
- Experimentally realistic boundary conditions
  - No-slip velocity boundary conditions
  - Thermal boundary conditions can be conducting or insulating

Turbulent Regime, $\sigma = 0.4$

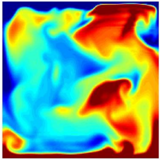
Temperature plot, vertical slice at $x = 0$



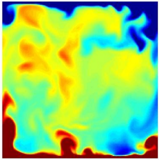
$R = 1 \times 10^6$



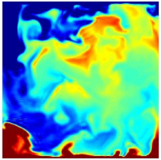
$R = 1 \times 10^7$



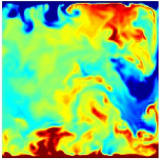
$R = 2 \times 10^7$



$R = 4 \times 10^7$



$R = 1 \times 10^8$

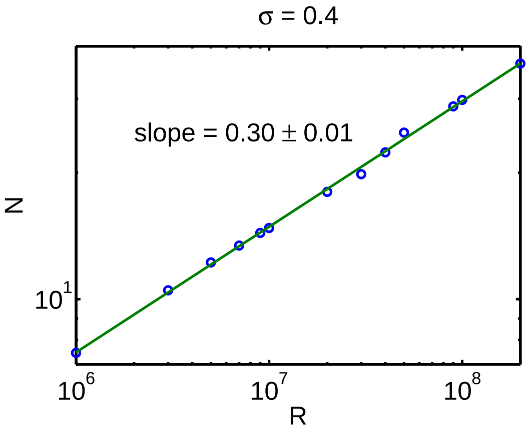


$R = 2 \times 10^8$

Dependence of Heat Transport on Rayleigh number

Heat transport → Nusselt number N.

$$N = N_0 R^{\gamma}$$



Comparisons of scaling of Nusselt number to other results

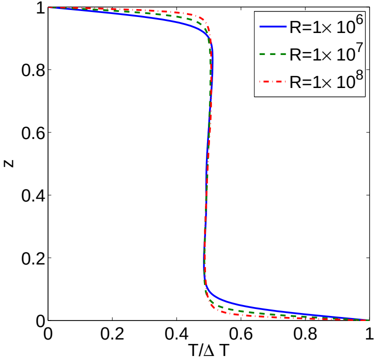
Numerical:

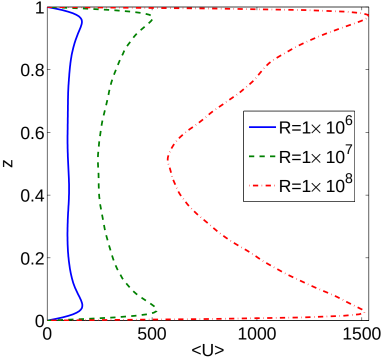
Group	σ	Range of R	Γ	γ
Scheel	0.4	$10^6 - 10^8$	1	0.30 ± 0.01
Emran(JFM 2008)	0.7	$10^7 - 10^9$	1	0.283
Amati(Phys Fluds 2005)	0.7	$10^6 - 10^{14}$	1/2	1/3
Johnston (PRL 2009)	1	$10^6 - 10^{10}$	2	0.285

Experimental:

Group	σ	Range of R	Γ	γ
Castaing (JFM 1989)	0.7-1	$10^7 - 10^{12}$	1	0.282 ± 0.006
Chavanne (PRL 1997)	0.6-0.73	$10^7 - 10^{11}$	1/2	2/7
Cioni (JFM 1997)	0.025	$10^6 - 10^8$	1	0.26 ± 0.02
Nikalenko (JFM 2005)	4.4	$10^9 - 10^{12}$	$.43 - .98$	0.309
Lui (PRE 1998)	$\simeq 7$	$10^8 - 10^{10}$	1	0.28 ± 0.06

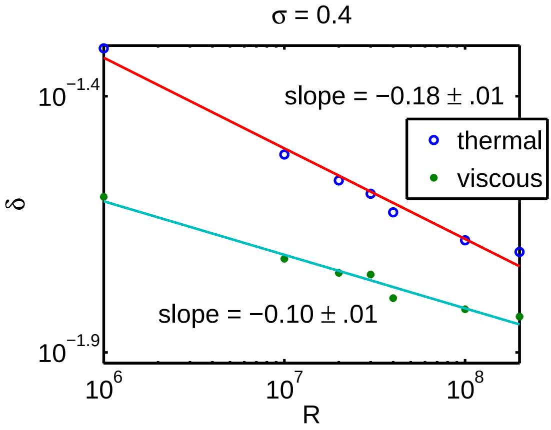
Dependence of boundary layer thickness on Rayleigh no.





Dependence of boundary layer thickness on Rayleigh no.

$$\delta = \delta_0 R^{\beta}$$



—Intersection at $R \simeq 1 \times 10^{10}$
—Chavanne (PRL 1997) sees transition to ultimate regime at $R \simeq 10^{11}$ for $\sigma = 0.7$
—Cionni (JFM 1997) sees at $R \simeq 10^{9}$ for $\sigma = 0.025$
—Xin (PRL 1996) sees same behavior but thermal $<$ viscous so no crossover for $\sigma \simeq 7$.

Comparisons of scaling of thermal boundary layer to other results

Numerical:

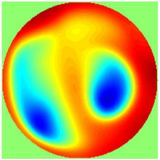
Group	σ	Range of R	Γ	β (thermal)	β (viscous)
Scheel	0.4	$10^6 - 10^8$	1	-0.18 ± 0.01	-0.10 ± 0.01

Experimental:

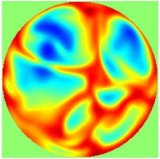
Group	σ	Range of R	Γ	β (thermal)	β (viscous)
Lui (PRE 1998)	$\simeq 7$	$10^8 - 10^{10}$	1	-0.285 ± 0.04	
Xin (PRL 1996)	$\simeq 7$	$10^8 - 10^{10}$	1	-0.29	-0.16
Sun (JFM 2008)	4.3	$10^9 - 10^{10}$	1	-0.32 ± 0.05	-0.27
Belmonte (PRE 1994)	0.7	$10^5 - 10^{11}$	1	-0.29 ± 0.01	
Du Puits (JFM 2007)	0.7	$10^9 - 10^{12}$	1	-0.25	

Boundary Layers, $\sigma = 0.4$

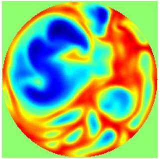
Temperature plot, horizontal slice at $z = \delta/2$, $\delta =$ boundary layer thickness, bottom boundary layer



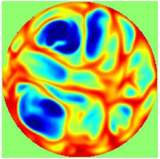
$R = 1 \times 10^6$



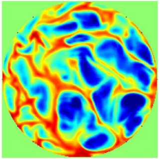
$R = 1 \times 10^7$



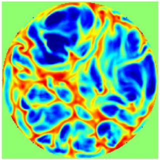
$R = 2 \times 10^7$



$R = 4 \times 10^7$



$R = 1 \times 10^8$



$R = 2 \times 10^8$

Conclusions

- Can run simulations up to $R = 2 \times 10^8$
- See large-scale circulation, orientational drift, still looking for reversals
- Heat transport scaling: $N = N_0 R^{0.30 \pm .01}$, closer to $2/7$ than $1/3$, but somewhat unconclusive.
- Thermal boundary layer scaling: $\delta_{\rm th} = \delta_{\rm th0} R^{-0.18 \pm 0.01}$, viscous boundary layer scaling: $\delta_{\rm v} = \delta_{\rm v0} R^{-0.11 \pm 0.01}$.
- Viscous boundary layer is thinner than thermal boundary layer. Since it's scaling exponent is smaller, expect a crossover at $R \simeq 10^{10}$ for $\sigma = 0.4$

Thanks to Research Corporation